

## Rules for Plotting Root-locus Diagrams (Negative Feedback)

In the following rules  $n \geq m$ , where  $n$  is the number of poles and  $m$  is the number of zeros.

**Rule 1**    **The number of loci** or branches is equal to the number of open-loop poles,  $n$ .

**Rule 2**    **The root loci begin** at open-loop poles and terminate at open-loop zeros. The termination of  $(n - m)$  of the loci will occur at the zeros at infinity along asymptotes to be described later.

In the case of a  $q^{\text{th}}$ -order pole,  $q$  loci emerge from it. For a  $q^{\text{th}}$ -order zero,  $q$  loci terminate there. A pole  $p_a$  of order  $q$  is present in the open-loop transfer function if the denominator of  $G$  contains  $(s - p_a)^q$ . A zero  $z_a$  of order  $q$  is present if the numerator of  $G$  contains  $(s - z_a)^q$ .

**Rule 3**    **Locus on real axis.** The real axis is part of the root locus when the sum of the number of poles and zeros to the right of a point on the real axis is odd. It is necessary to consider only the real poles and zeros in applying this rule, for the complex poles and zeros always occur in conjugate pairs and their effects cancel in checking the angle criterion for points on the real axis. Furthermore, a  $q$ th-order pole (or zero) must be counted  $q$  times in applying the rule.

**Rule 4**    **Asymptotes.** There are  $(n - m)$  loci which approach (as  $K \rightarrow \infty$ ) asymptotically  $(n - m)$  straight lines, radiating from the center of gravity of the poles and zeros of the open-loop transfer function. The center of gravity is given by

$$\gamma = \frac{\sum_{j=1}^n p_j - \sum_{i=1}^m z_i}{n - m}$$

These asymptotic lines make angles of:

$$\pi \left( \frac{2k + 1}{n - m} \right)$$

with the real axis and are, therefore, equally spaced at angles  $2\pi/(n - m)$  to each other

$$k = 0, 1, 2, \dots, n - m - 1.$$

**Rule 5**    **Breakaway point.** The point at which two root loci, emerging from adjacent poles (or moving toward adjacent zeros) on the real axis, intersect and then leave (or enter) the real axis is determined by the solution of the equation

$$\sum_{i=1}^m \frac{1}{s - z_i} = \sum_{j=1}^n \frac{1}{s - p_j}$$

These loci leave (or enter) the real axis at angles of  $\pm \pi/2$ . The above equation is solved by trial by checking it for various test points,  $s = s_c$ , on the real axis between the poles (or zeros) of interest. For real poles or zeros, the terms in the denominator of the equation are obtained by simply measuring distances along the real axis between the test

point and the poles and zeros. If a pair of complex poles,  $p_i = a_i \pm jb_i$ , are present, add to the right side of the equation the term

$$\frac{2(s - a_i)}{(s - a_i)^2 + b_i^2}$$

(This term accounts for both poles of the complex pair.) This term is merely the result of simplifying the sum

$$\frac{1}{s - a_i - jb_i} + \frac{1}{s - a_i + jb_i}$$

For a pair of complex zeros, add a similar term to the left-hand side of the equation.

**Rule 6** **Angle of departure or approach.** There are  $q$  loci emerging from each  $q^{\text{th}}$ -order open-loop pole at angles determined by

$$\theta = \frac{1}{q} \left[ (2k + 1)\pi + \sum_{i=1}^m \angle(p_a - s_z) - \sum_{\substack{j=1 \\ j \neq a}}^n \angle(p_a - p_j) \right]$$

$$k = 0, 1, 2, \dots, q - 1$$

where  $p_a$  is a particular pole of order  $q$ . Each of the  $m$  loci which do not approach the asymptotes will terminate at one of the  $m$  zeros. They will approach their particular zeros at angles

$$\theta = \frac{1}{v} \left[ (2k + 1)\pi + \sum_{j=1}^n \angle(z_b - p_j) - \sum_{\substack{i=1 \\ i \neq a}}^m \angle(z_b - z_i) \right]$$

$$k = 0, 1, 2, \dots, v - 1$$

where  $z_b$  is a particular zero of order  $v$ . For simple poles (or zeros) on the real axis, the angle of departure (or approach) will be 0 or  $\pi$

from: Coughanowr, D.R. and Koppel, L.B., *Process Systems Analysis and Control*, McGraw-Hill Book Company, 1965.

RLOCUS Evans root locus.

For a SISO system SYS with numerator N(s) and denominator D(s), RLOCUS(SYS) calculates and plots the locus of the roots of

$$H(s) = D(s) + k * N(s) = 0$$

for a set of positive gains K which are adaptively calculated to produce a smooth plot.

RLOCUS(SYS,K) uses a user-specified vector K of gains.

[R,K] = RLOCUS(SYS) or R = RLOCUS(SYS,K) returns the matrix R of complex root locations for the gains K. R has LENGTH(K) columns and its j-th column lists the closed-loop roots for the gain K(j).

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TF Specify transfer functions or convert LTI model to transfer function.

TFSYS = TF(SYS) converts an arbitrary LTI model SYS to transfer function format. The output TFSYS is a TF object.

TFSYS = TF(SYS,'inv') uses a fast algorithm for state-space SYS, but is typically less accurate for high-order systems.

You can create SISO or MIMO transfer functions (TF) by

SYS = TF(NUM,DEN)	Continuous-time system NUM(s)./DEN(s)
SYS = TF(NUM,DEN,T)	Discrete-time system NUM(z)./DEN(z) with sampling time T (Set T=-1 if undetermined)
SYS = TF	Default empty TF object
SYS = TF(M)	Static gain matrix
SYS = TF(NUM,DEN,LTI)	Transfer function with LTI properties inherited from system LTI (can be SS, TF, or ZPK).

All the above syntaxes may be followed by Property/Value pairs.

(Type "help ltiprops" for details on assignable properties).

The output SYS is a TF object.

The default TF variables 's' and 'z' can be altered by resetting the 'Variable' property. Alternatives include 'p' for continuous TF and 'z^-1' or 'q' for discrete TF.

For SISO systems, NUM and DEN are row vectors listing the numerator and denominator coefficients in

- \* descending powers of s or z by default
- \* ascending powers of q = z^-1 if 'Variable' is set to 'z^-1' or 'q' (DSP convention).

Example:

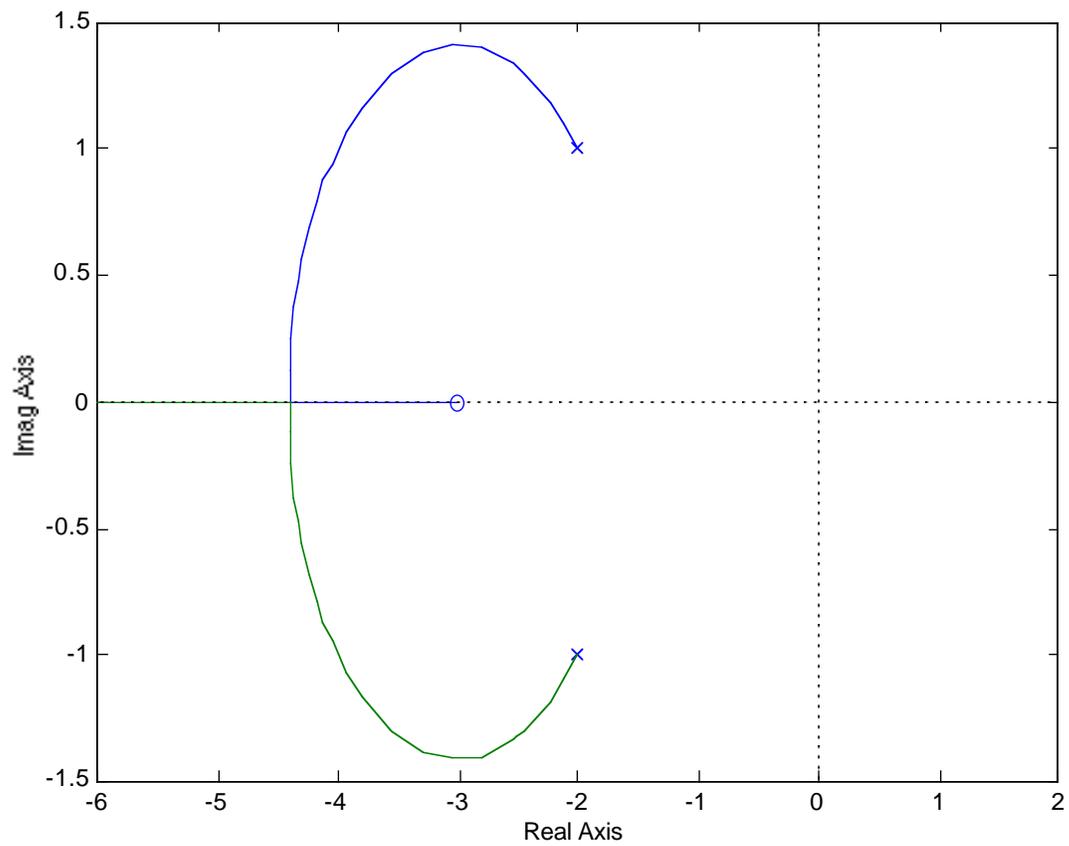
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»h=tf([0.3333333 1],[1 4 5])
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Transfer function:

$$0.3333 s + 1$$
$$\frac{0.3333 s + 1}{s^2 + 4 s + 5}$$

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»rlocus(h)
```

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»
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A few examples for you to study with:

For the Transfer Function:

$$\frac{s^1 + 3}{s^3 + 2s^2 + 2s^1}$$

K=inf point:  $-3 + 0i$

K=0 point:  $0 + 0i$

K=0 point:  $-1 + 1i$

K=0 point:  $-1 - 1i$

K>0 asymptote angles: 90 270

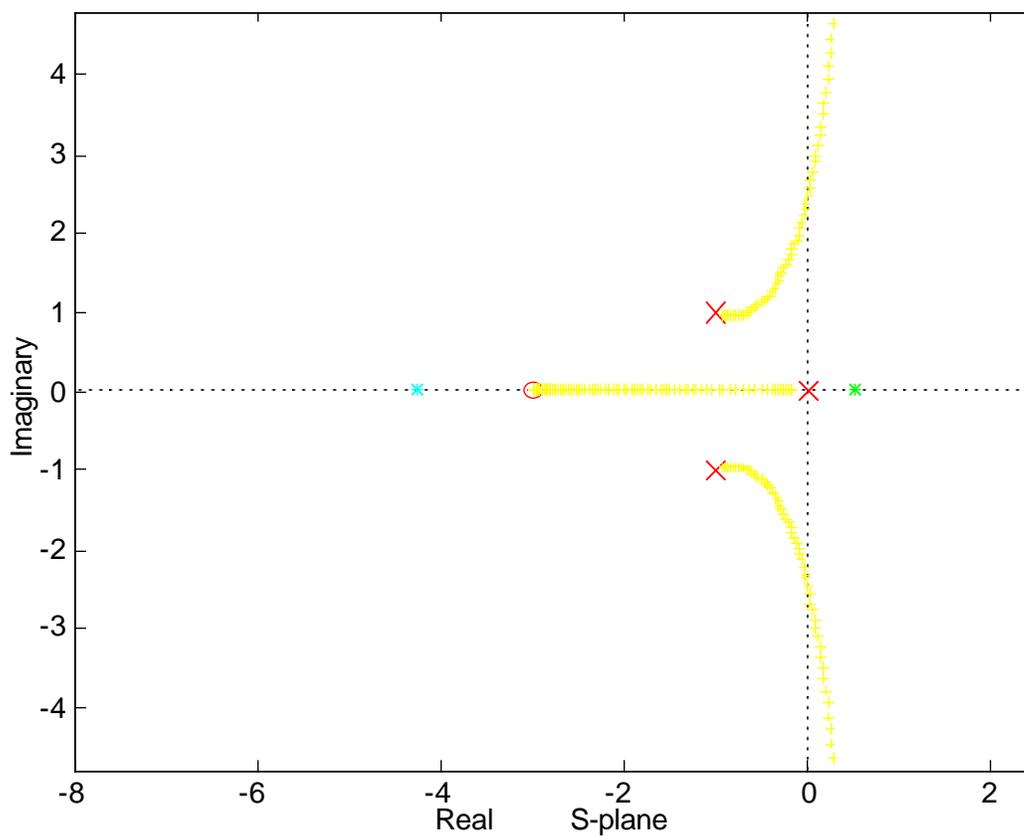
K<0 asymptote angles: 0 180

Asymptote intersection: 0.5

K>0 real axis locus: (0 to -3)

K>0 Departure angle =  $-18.43$  degrees at Pole:  $-1 + 1i$

For K =  $-39.31$  a Breakaway point appears at:  $-4.256 + 0i$



For the Transfer Function:

$$\frac{s^1 + 2}{s^2 + 1.5s^1 + 0.5}$$

K=inf point: -2 + 0i

K=0 point: -1 + 0i

K=0 point: -0.5 + 0i

K>0 asymptote angles: 180

K<0 asymptote angles: 0

Asymptote intersection: 0

Press any key to continue...

K>0 real axis locus: (-0.5 to -1), (-2 to -Inf)

Only trivial angles of departure/arrival exist.

For K = 4.949 a Breakaway point appears at: -3.225 + 0i

