Population Dynamics, Positive Feedback, & Building Equations in Simulink

Population Behavior

- Populations do not, generally, only grow by colonization or die to extinction.
- Populations tend to oscillate somewhere above levels which risk extinction and that at which their habitat would be destroyed.

Goals of Population Modeling

- Quantify the rates of birth, death immigration, and emigration.
- Use this information to explain what is influencing the timing and magnitude of these fluctuations.
- Alter the mean level of these fluctuations.
- Prevent over exploitation and extinction.

General Approach to Population Modeling

- Start simple
- Add complexity when
  - The current model does not include needed components (e.g. the ones we're studying or ones of known importance)
  - The current output does not match the behavior of the real system (e.g. population growth does not match real population)
- Stop when additional complexity is not *required*

Two General Classes of Model

- Organisms with overlapping generations
  - Humans, Bacteria, Protozoans, Birds, Mammals, Trees, and Some Insects.
- Organisms having discrete generations.
  - Annual plants, Moths & Butterflies (eggs are laid by females in phase, Caterpillars hatch. . . Eggs are laid only by new generation.

Exponential Growth

$$\frac{dN}{dt} = r \bullet N$$

Overlapping Generations Exponential Growth

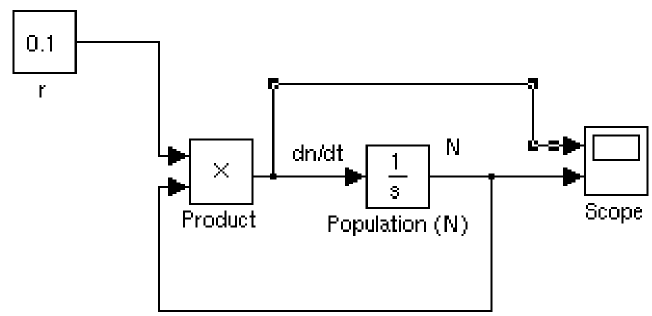
- Where:
  - N = the number of individuals in the population
  - t = time
  - r = intrinsic rate of natural increase
  - = birth rate - death rate under fixed conditions
- The coefficient "r" is a function of:
  - Reproductive delay
  - Distribution of progeny during the organism's lifespan
  - Length of life

Overlapping Generations Exponential Growth

$$\frac{dN}{dt} = r \bullet N$$

Exponential Growth Simulink

$$\frac{dN}{dt} = r \bullet N$$



Malthusian Growth

- Thomas Malthus (b. 1766; d. 1834)
- *Essay on the Principle of Population*
- Predicted disaster noting that, unless offset by war or disease, the world population grows at an exponential rate (doubling every 25 years) while food supply grows linearly.

Logistic Growth

- Populations do not expand exponentially forever.
- There is a limit to the number of individuals that a space can support.
  - Limit is known as “k”, the carrying capacity.
- The rate of growth is reduced based on the space available for individuals.

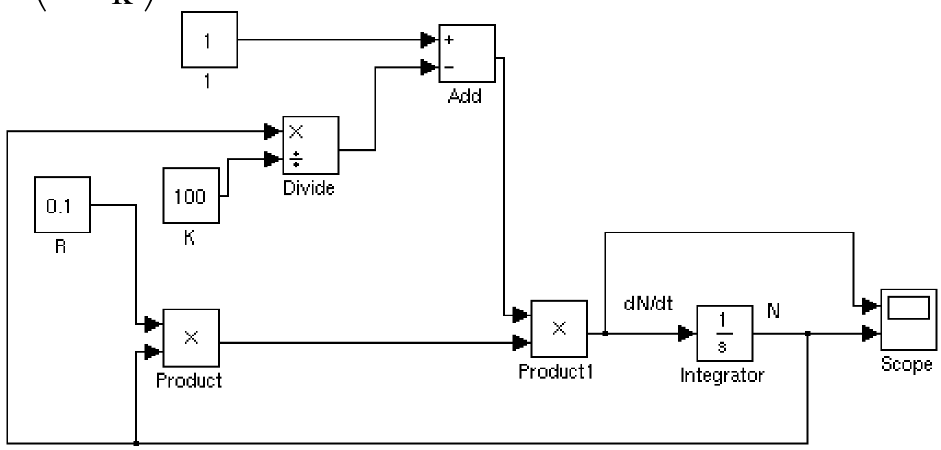
Logistic Growth Carrying Capacity

$$\left(1 - \frac{N}{k}\right)$$

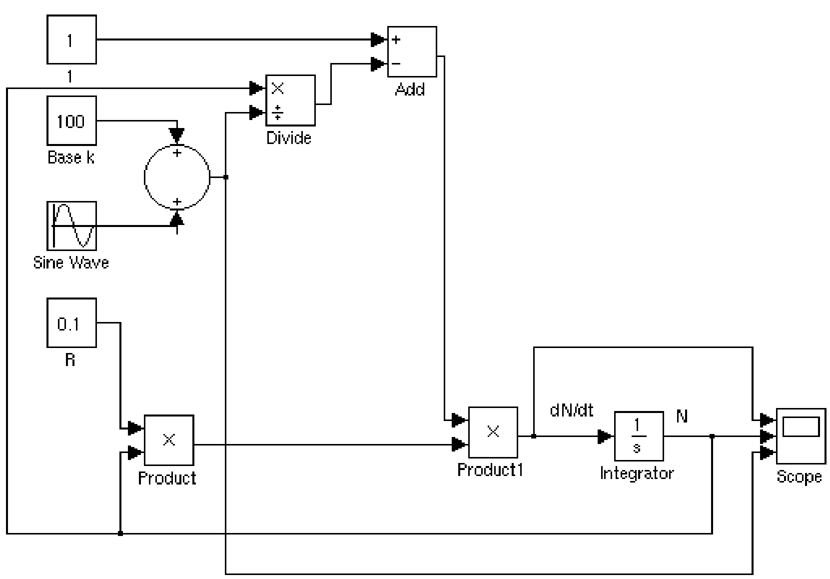
- The above term is added to the equation
  - $N < k$ growth rate is postitive
  - $N = k$ growth rate is 0
  - $N > k$ growth rate is negative

Loqistic Growth Equation & Simulink

$$\frac{dN}{dt} = r \bullet N \bullet \left(1 - \frac{N}{k}\right)$$



Logistic Growth Variable Carrying Capacity



Logistic Growth Variable Carrying Capacity

$$\frac{dN}{dt} = r \bullet N \bullet \left(1 - \frac{N}{100 + 10 \bullet \text{sine}\,(2 \bullet \pi \bullet t)}\right)$$

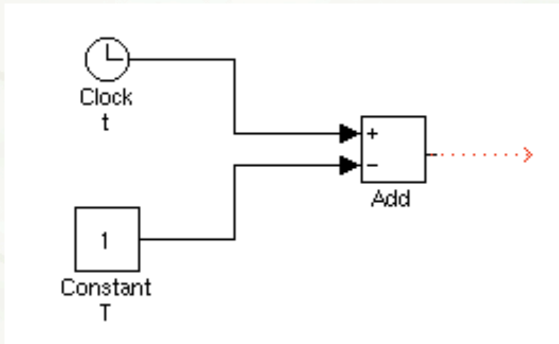
Logistic Growth Time Lag

- Density dependence is not usually instantaneous.
- Due or organism generation time or environmental recovery.

$$\frac{dN}{dt} = r \bullet N \bullet \left(\frac{k - (t - T)}{k}\right)$$

- Where: T = Time lag

Logistic Growth
Time Lag Term & Simulink



Logistic Growth
Variable Carrying Capacity & Time Lag

$$\frac{dN}{dt} = r \bullet N \bullet \left(\frac{(100 + 10 \bullet \text{sine}\,(2 \bullet \pi \bullet t)) - (t - T)}{100 + 10 \bullet \text{sine}\,(2 \bullet \pi \bullet t)} \right)$$

- ***This is why we use numerical integration routines***