Laplace transform pairs

	f(t)	F(s)
1	Unit impulse, δ (t)	1
2	Unit step, 1 (t)	$\frac{1}{s}$
3	Unit ramp, t	$\frac{1}{s^2}$
4	n^{th} – Order ramp, t^n	$\frac{n!}{s^{n+1}}$
5	$\frac{t^{n-1}}{(n-1)\,!}$ (n = 1, 2, 3,…)	$\frac{1}{s^n}$
6	Exponential e^{-at}	$\frac{1}{s+a}$
7	n^{th}-Order exponential, $t^n\, e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
8	te^{-at}	$\frac{1}{(s+a)^2}$
9	$\frac{1}{(n-1)!} t^{n-1} e^{-at}$ (n = 1, 2, 3…)	$\frac{1}{(s+a)^n}$
10	$\frac{1}{b-a}\left(e^{-at} - e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
11	$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at} - ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
12	sin ωt	$\frac{\omega}{s^2+\omega^2}$
13	cos ωt	$\frac{s}{s^2+\omega^2}$
14	Damped sine, e^{-at} sin ωt	$\frac{\omega}{(s+a)^2+\omega^2}$
15	Damped cosine, e^{-at} cos ωt	$\frac{s+a}{(s+a)^2+\omega^2}$
16	Diverging sine, t sin bt	$\frac{2bs}{(s^2+b^2)^2}$
17	Diverging cosine, t cos bt	$\frac{s^2-b^2}{(s^2+b^2)^2}$
18	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
19	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t-\phi\right)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2+2\zeta\,\omega_n s+\omega_n^2}$

20	$-\frac{1}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t} \sin\left(\omega_n \sqrt{1-\zeta^2}\, t + \phi\right) \qquad \phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$

Properties of Laplace transforms

1	$\mathscr{L}_{\pm}\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0\pm)$
2	$\mathscr{L}_{\pm}\left[\frac{d}{dt^2} f(t)\right] = s^2 F(s) - s f(0\pm) - f(0\pm)$
3	$\mathscr{L}_{\pm}\left[\frac{dn}{dt^n} f(t)\right] = s^n F(s) - \sum_{k=1}^{n} s^{n-k} \overset{(k-1)}{f}(0\pm) \qquad \text{where } \overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}} f(t)$
4	$\mathscr{L}_{\pm}\left[\int f(t)dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)dt\right]_{t=0\pm}$
5	$\mathscr{L}\left[\int_0^t f(t)dt\right] = \frac{F(s)}{s}$
6	$\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s) \text{ if } \int_0^\infty f(t)\,dt \text{ exists}$
7	$\mathscr{L}\left[e^{-at} f(t)\right] = F(s+a)$
8	$\mathscr{L}\left[f(t-a)1(t-a)\right] = e^{-as} F(s) \quad a \geq 0$
9	$\mathscr{L}\left[t\, f(t)\right] = -\frac{dF(s)}{ds}$
10	$\mathscr{L}\left[t^2 f(t)\right] = \frac{d^2}{ds^2} F(s)$
11	$\mathscr{L}\left[t^n f(t)\right] = (-1)^n \frac{d^n}{ds^n} F(s) \qquad n = 1, 2, 3, \ldots$
12	$\mathscr{L}\left[\frac{1}{t} f(t)\right] = \int_s^\infty F(s)\,ds \text{ if } \lim_{t\to 0} \frac{1}{t} f(t) \text{ exists}$
13	$\mathscr{L}\left[f\left(\frac{t}{a}\right)\right] = aF(as)$
14	$\mathscr{L}\left[\int_0^t f_1(t-t) f_2(t)dt\right] = F_1(s)\, F_2(s)$
15	$\mathscr{L}\left[f(t)g(t)\right] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s-p)\,dp$