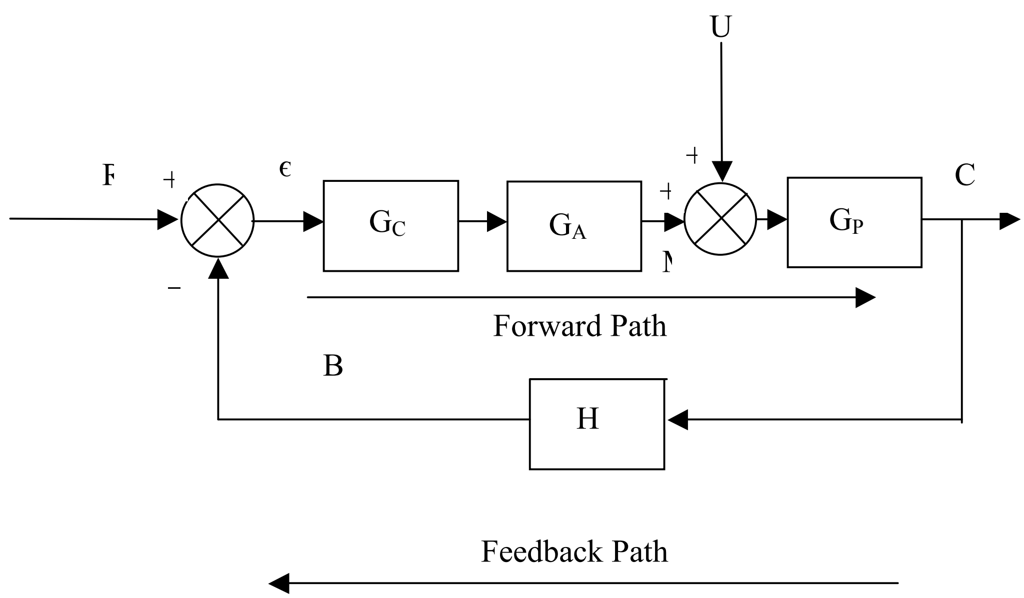
Block Diagram Manipulations

The Basic Control Loop

In this section we will examine methods for simplifying systems of transfer functions to a single function. As the first example, we will consider a generic control loop. In this system, G_C represents a control algorithm. G_A is the transfer function for the actuator. The function of an actuator is to take the output of the controller (usually a low power electrical signal) and convert it to something that can alter the process; for example, a heater or an agitator. G_P is the transfer function for the process. H is the transfer function for the sensor (e.g., a thermometer).

This system has two inputs and one output. *R* is the setpoint (or desired output) for the system. When the output (via the sensor) is subtracted from the setpoint, the resulting error, ε , is used as the input
for the controller. The second input, *U*, is the load or disturbance input. This input is usually not under the engineer's control. Some examples of disturbances might be feedstock entering a reactor or external temperature changes. The output variable, *C*, is the variable being controlled.

In general, there are two categories of control problems. In "Servo" problems, there is no change in load, but the setpoint changes. In "Regulator" problems, the setpoint remains constant, but the load changes. In real systems both the load and the setpoint may change simultaneously.



R = Setpoint
ε = Error
U = Load / Disturbance Variable
B = Variable Produced by Measuring Element
C = Controlled Variable
M = Manipulated Variable

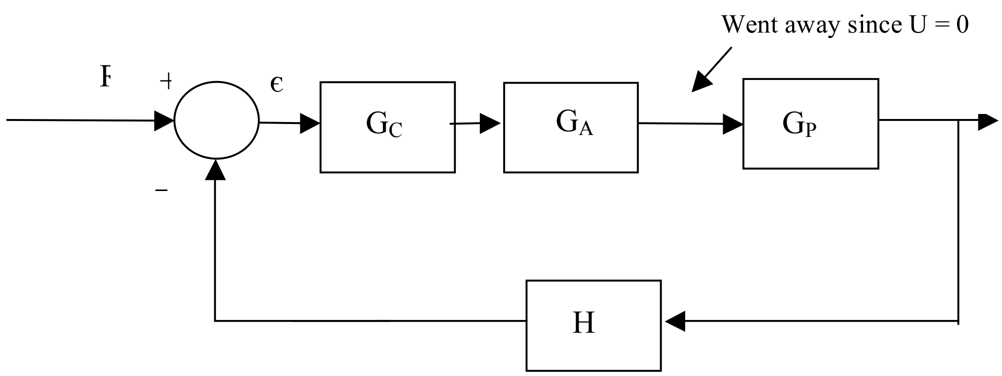
G_c = Transfer Function of Controller
G_A = Transfer Function of Actuator
G_p = Transfer Function of Process
H = Transfer Function of Measuring Element

---

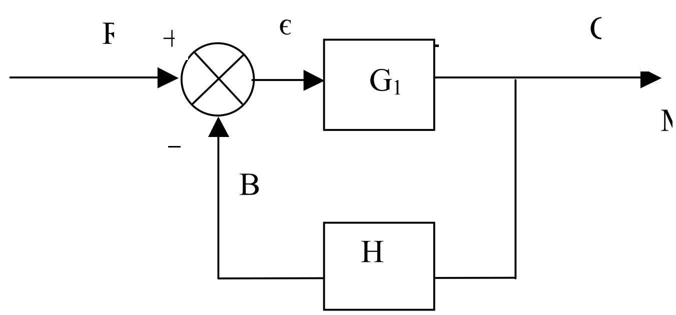
1. Determining transfer function for relating R to C, the servo problem.

   A. These are "overall" transfer functions (i.e., include entire process), so we are looking for C/R.

   B. Remember, these are differences from steady state, not absolute values. If the only change is in R, U = 0.



Then, by combining $G_C, G_A, and\ G_P$ into a single transfer function "G_1" ($G_1 = G_C\ G_A\ G_P$).



This loop can be solved using three simultaneous equations:

$$C = G_1 \varepsilon \qquad \text{I}$$

$$B = HC \qquad \text{II}$$

$$\varepsilon = R - B \qquad \text{III}$$

III into I yields

$$C = G_1 (R - B)$$

Substitute in II

$$C = G_1 (R - HC)$$

$$C = G_1R - HG_1C$$

$$C + HGC = G_1R$$

$$C = \frac{G_1 R}{1 + G_1 H}$$

$$\frac{C}{R} = \frac{G_1}{1 + G_1 H}$$

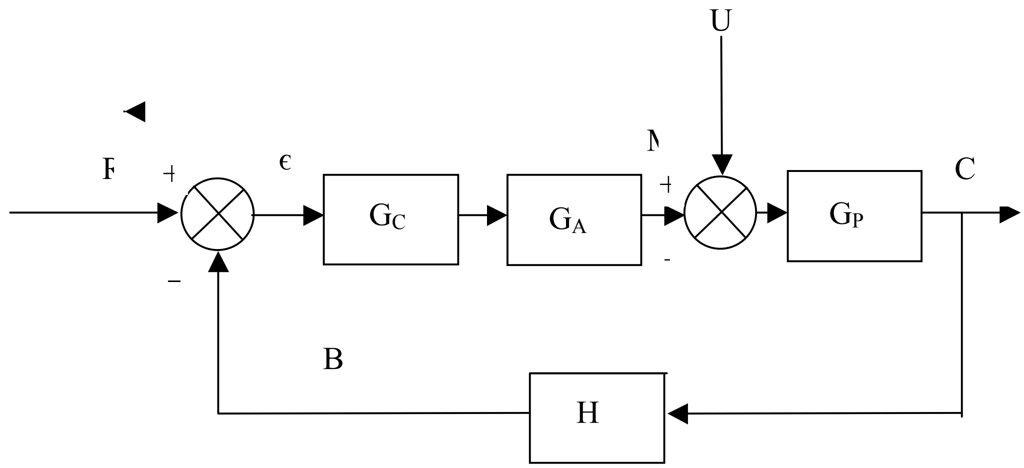
$$= \frac{G_C G_A G_P}{1 + G_C G_A G_P H}$$

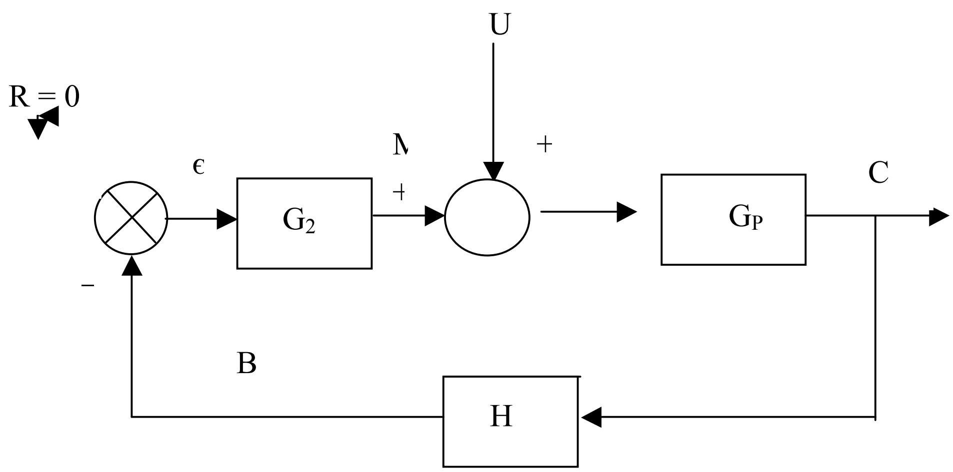
---

2. Determining the transfer function relating *U* to *C,* the regulator problem.

   A. As before, we are looking for the overall transfer function relating *C* to *U*. So the goal is C/U .

   B. In this case, *U* is changing, not *R*. Therefore, *R* is 0 in deviation terms.





$$G_2 = G_C \ G_A$$

$$C = (U + M)G_P$$

$$M = G_2 \varepsilon$$

$$\varepsilon = -B$$

$$B = HC$$

Then

$$C = (U = G_2 \varepsilon)G_P$$

$$C = (U + G_2(-B))G_P$$

$$C = (U + G_2(-HC))G_P$$

$$C = UG_P - G_2 G_P HC$$

$$C + G_2 G_P HC = UG_P$$

$$C(1 + G_2 G_P HC) = UG_P$$

$$C = U \frac{G_P}{1 + G_2 G_P H}$$

$$C = U \frac{G_p}{1 + G_C G_A G_P H}$$

$$\frac{C}{U} = \frac{G_P}{1 + G_C G_A G_P H}$$

If changes occur in both *R* and *U*, then we can solve the system by adding the two solutions together.

So, if

$$\frac{C}{R} = \frac{G_C\, G_A\, G_P}{1 + G_C\, G_A\, G_P\, H}$$

And

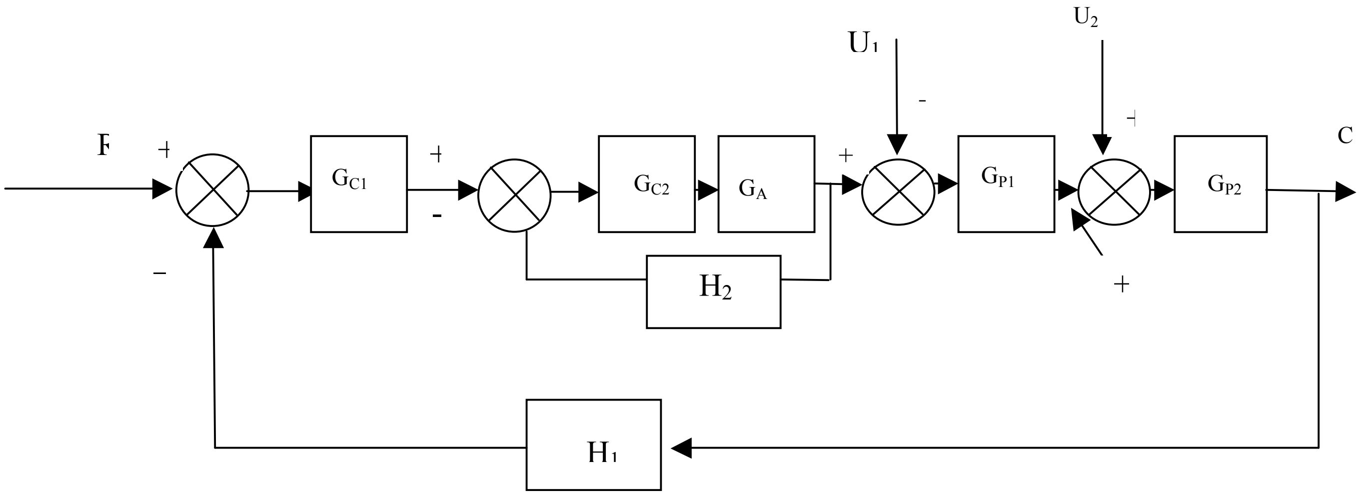
$$\frac{C}{U} = \frac{G_P}{1 + G_C\, G_A\, G_P\, H}$$

The response of the system to both changes is

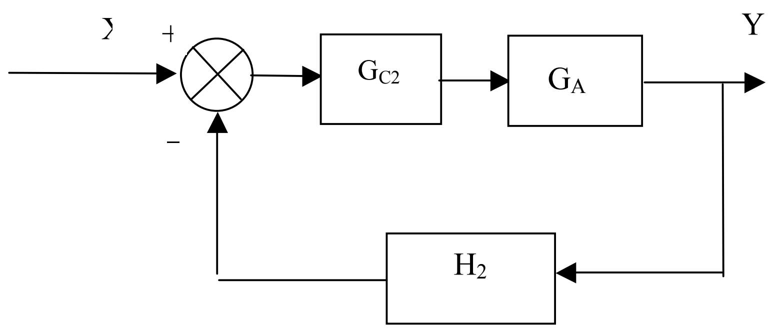
$$C = R\left(\frac{G_C\, G_A\, G_P}{1 + G_C\, G_A\, G_P\, H}\right) + U\left(\frac{G_P}{1 + G_C\, G_A\, G_P\, H}\right)$$

Or

$$C = \frac{R\, G_C\, G_A\, G_P + U\, G_P}{1 + G_C\, G_A\, G_P\, H}$$



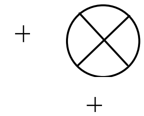
Start by eliminating the innermost loop.



This has the transfer function

$$\frac{Y}{X} = \frac{G_{C2}G_A}{1 + G_{C2}G_A H_2} = G\alpha$$

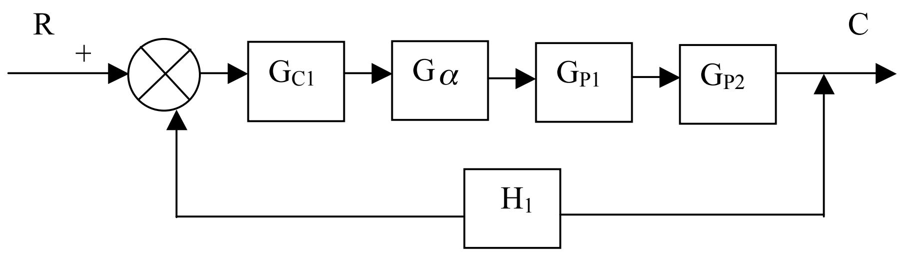
Note: What if this was a positive feedback loop; i.e.,



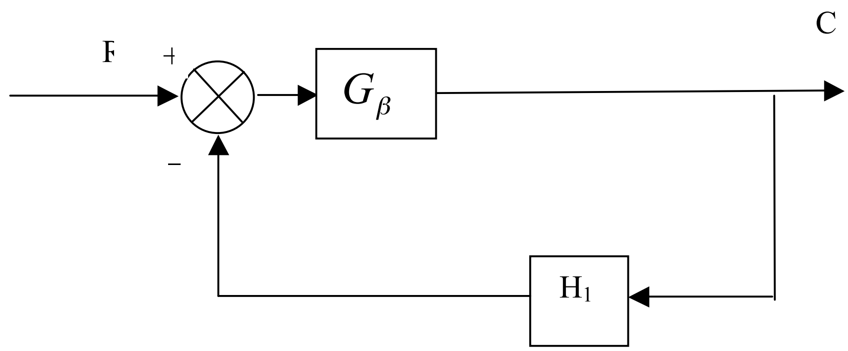
Then transfer function

$$\frac{Y}{X} = \frac{G_{C2}G_A}{1 - G_{C2}G_A H_2}$$

Because we are solving the servo problem, U_1 and U_2 are 0. Now we have:



This becomes



Where

$$G_B = G_A G_\alpha G_{P1} G_{P2}$$

$$\frac{C}{R} = \frac{G_\beta}{1 + G_\beta H_1}$$

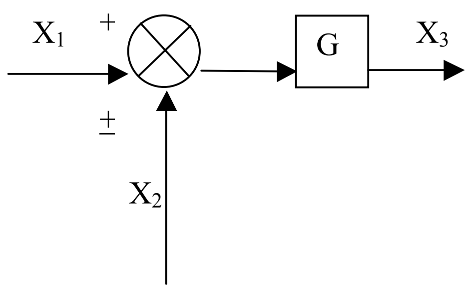
So

$$\frac{C}{R} = \frac{G_{C1} G_\alpha G_{P1} G_{P2}}{1 + G_{C1} G_\alpha G_{P1} G_{P2} H_1}$$

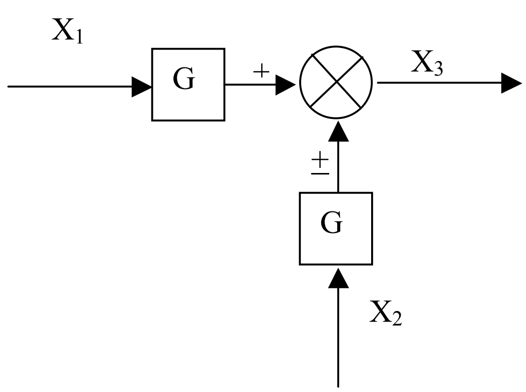
Some Additional Block Diagram Manipulation Techniques

In some cases, simplification of the block diagram will require altering the order of the various elements of a block diagram. Some of the possible operations are described below.

1. Moving a summing junction behind a block

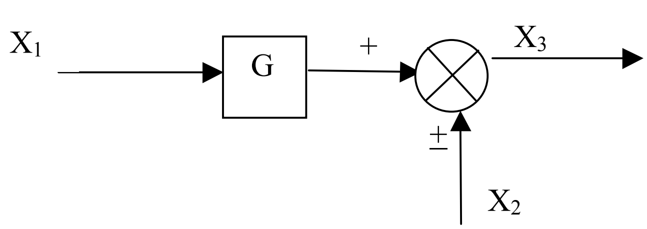


$$X_3 = G(X_1 \pm X_2)$$



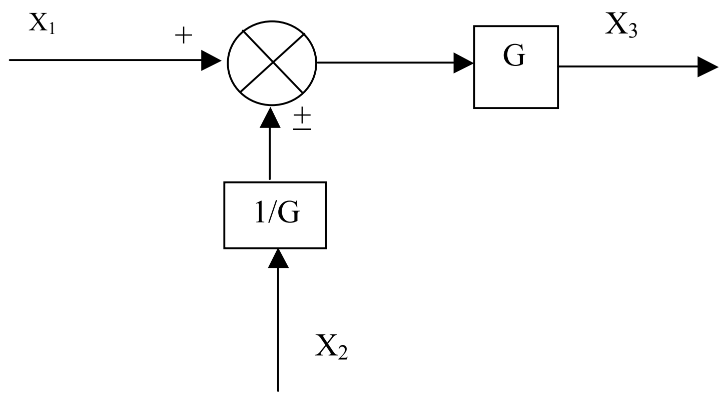
$$X_3 = GX_1 \pm GX_2$$

2. Moving a summing junction ahead of a block



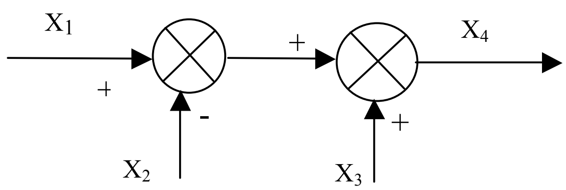
$$X_3 = GX_1 \pm X_2$$

is equivalent to



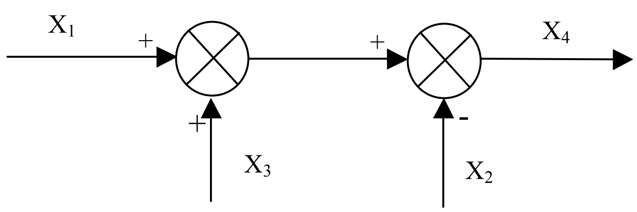
$$X_3 = (X_1 \pm (1/G)X_2)G$$
$$= GX_1 \pm 1/G\ X_2$$
$$= GX_1 \pm X_2$$

3. Altering the order of summing junctions



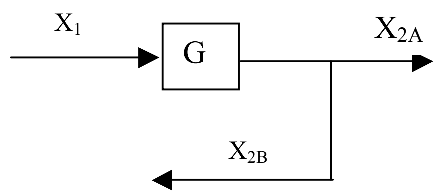
$$X_4 = (X_1 - X_2) + X_3$$
$$= X_1 - X_2 + X_3$$

Is equivalent to



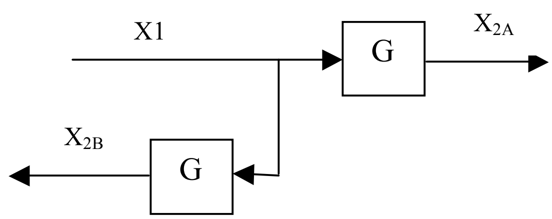
$$X_4 = (X_1 + X_3) - X_2$$
$$= X_1 - X_2 + X_3$$

4. Moving a pick off point ahead of a block



$$X_{2A} = G\,X_1$$

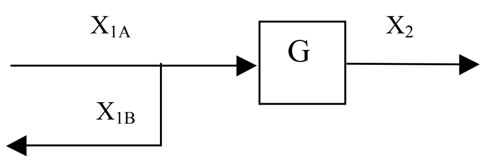
$$X_{2B} = G\,X_1$$

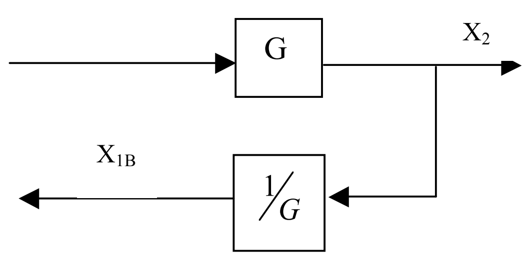


$$X_{2A} = G\,X_1$$

$$X_{2B} = G\,X_1$$

5. Moving a pick off point behind a block



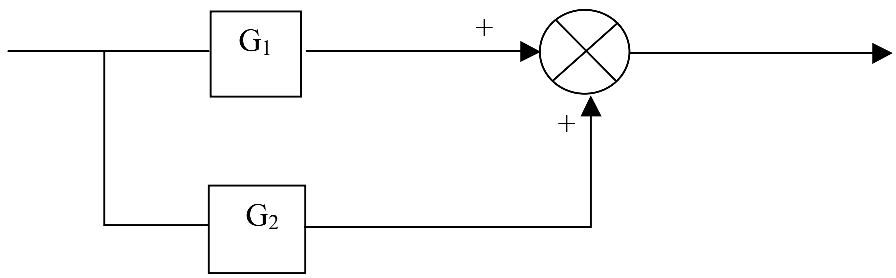


$$X_2 = G\,X_{1A}$$

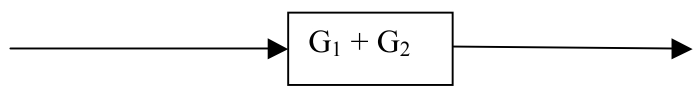
$$X_2 = G\,X_{1B}$$

$$X_{1B} = \frac{X_2}{G}$$

6. Parallel forward paths

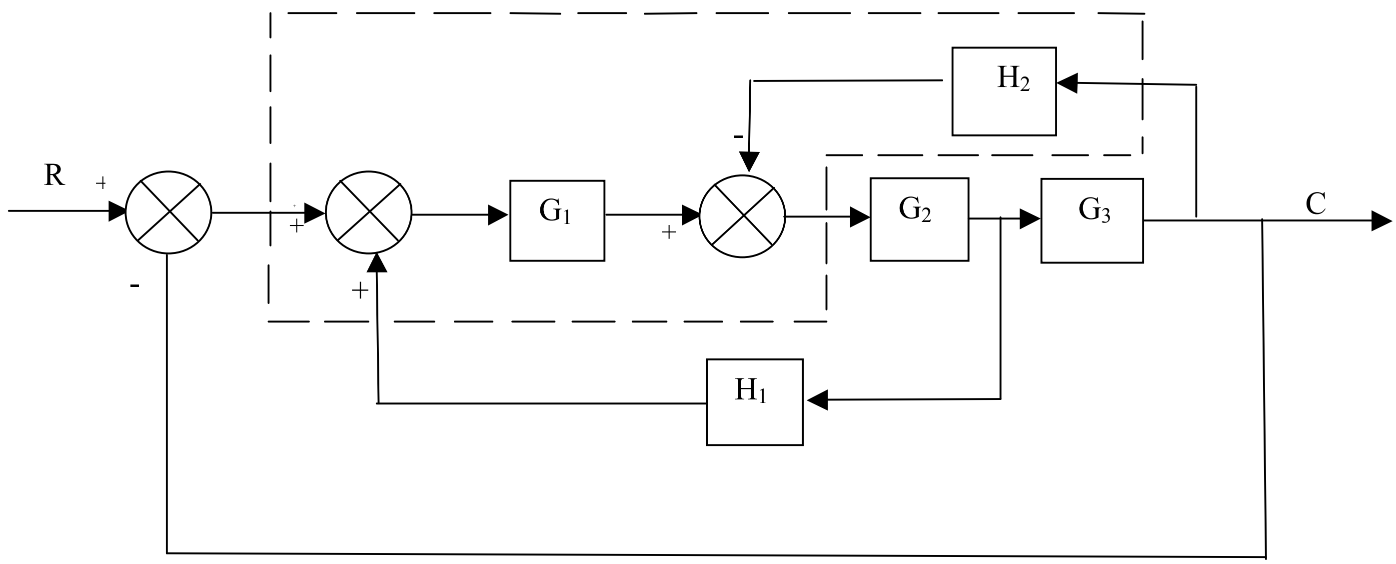


Becomes

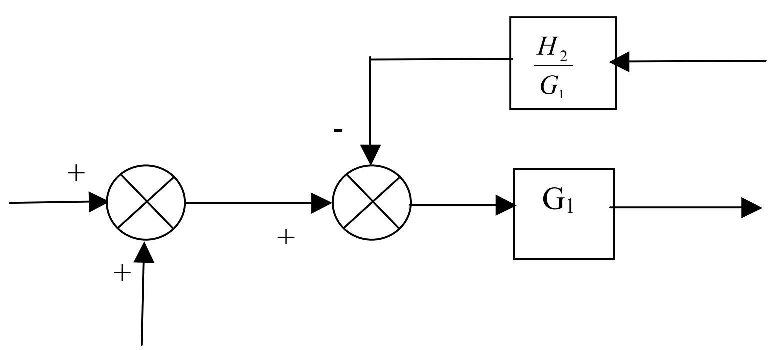


A Complex System with Interlaced Control Loops

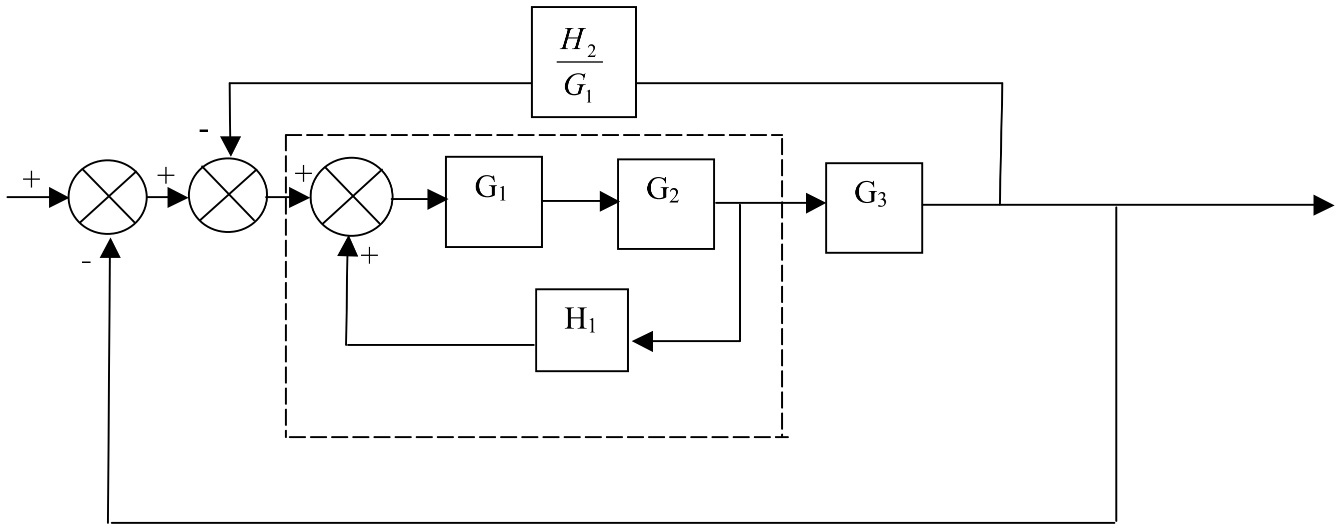
Here is another complex example to practice simplifying block diagrams. The challenging issue is the interlacing of the feedback loops. They need to be converted to nested loops.



In this case, we will manipulate the feedback loop containing H_2 to be outside the loop containing H_1. Then moving the summing junction before G_1, the highlighted area (indicated by dotted line) may be written:



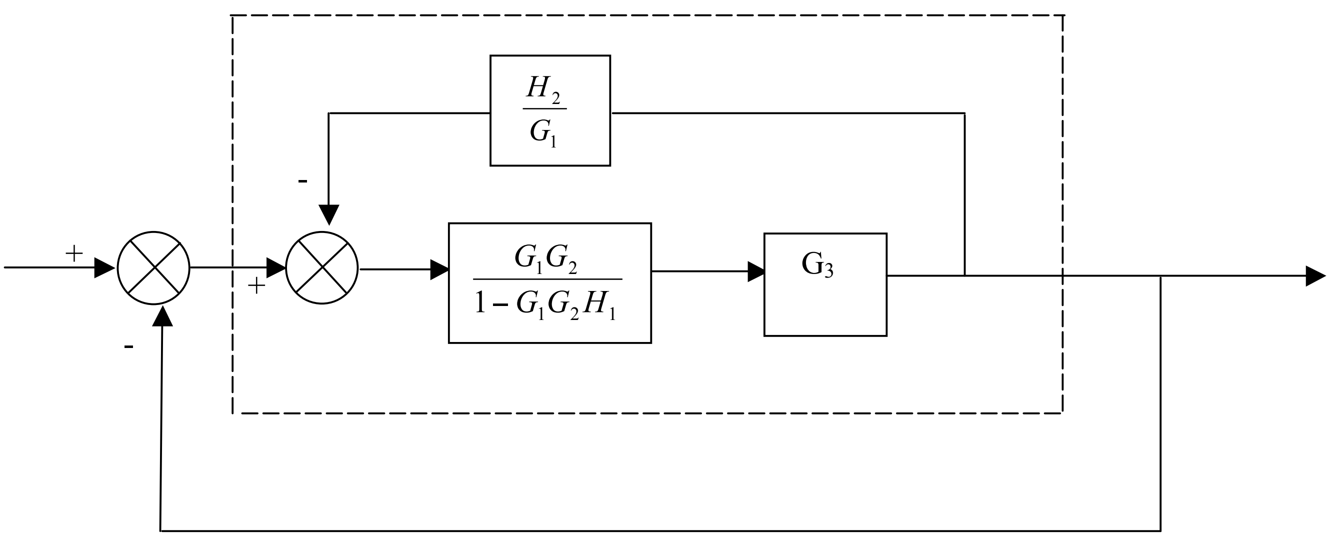
We can now swap the order of the summing junctions to yield



The transfer function for the boxed area is

$$\frac{G_1G_2}{1-G_1G_2H_1}$$

Note the sign change (since this is positive feedback loop)



Let

$$\frac{A}{B} = \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1}$$

Then the transfer function for the highlighted area is

$$\frac{A}{B} \bullet \frac{1}{1 + \frac{A}{B}\frac{H_2}{G_1}}$$

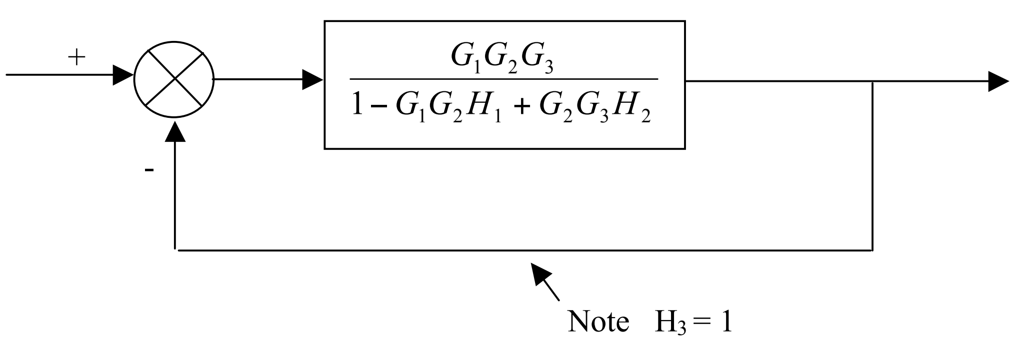
$$\frac{A}{B} \bullet \frac{1}{\frac{BG_1 + AH_2}{BG_1}}$$

$$\frac{A}{B} \bullet \frac{BG_1}{BG_1 + AH_2}$$

$$\frac{AG_1}{BG_1 + AH_2} = \frac{G_1 G_1 G_2 G_3}{G_1 (1 - G_1 G_2 H_1) + G_1 G_2 G_3 H_2}$$

$$= \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2} = \frac{D}{E}$$

And the diagram becomes



Then the transfer function becomes

$$\frac{\frac{D}{E}}{1+\frac{D}{E}}$$

So

$$\frac{\frac{D}{E}}{\frac{D+E}{E}}$$

So

$$\frac{D}{E} \bullet \frac{E}{D+E}$$

And

$$\frac{D}{D+E}$$

Substituting back in for *D* and *E*.

$$\frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}$$

Or

$$\longrightarrow \boxed{\frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3}} \longrightarrow$$